Auction theory

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Outline

Examples from the literature

— Cinque onze e sei!... Dico io!...
[...] *- Cinque onze e sei! - replicò il baronello senza dar retta, guardando in giro trionfante. - Cinque e quindici.*
[...] *- L'ultima offerta per le terre del comune! A sei onze la salma!... Uno! Due!...*
[Mastro Don Gesualdo, G. Verga]

Definition

An auction is a sale in which a good or service is sold to the highest (lowest) bidder

Examples

- luxury goods (art auctions)
- farm/fishing auctions
- public procurement (home refuse collection)
- highways management
- flexible electricity prices
- mobile phone coverage (spectrum auctions)
- emission allowances in Eu (global warming)
- school trips
- government bonds
- eBay
- keyword positions and advertisment
- oil fields

Outline

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Goals

- collect as much revenue as possible
- assign goods and services to the buyer that can make the best possible use of them
  - provide higher long term benefit for the society

Outline

Number of units

- Is it indivisible like a painting?
- Is it available in large numbers like government bonds?
- We will focus on single indivisible goods

Value

- Does it have the same value for all bidders (not known exactly by each of them: private information)?
  - signal of the common value
- Does it have an idiosyncratic value for each bidder (known by each of them)?
  - private value
- Some examples:
  - work of art
  - bonds
  - silver mine

Outline

2. Elements of an auction
   - Goals
   - Auctioned objects
   - Uncertainty
   - Format

Informations

- Is there asymmetric information?
- What infromation do the bidders know about the common value?
- What infromation do the bidders know about the private value?

Outline

2. Elements of an auction
   - Goals
   - Auctioned objects
   - Uncertainty
   - Format

English Auction

3 volunteers:

- an actioner
- bidder A
- bidder B

English Auction

Focus on an auctioner that wants to sell a painting:

- (s)he starts from the lowest acceptable price 100*USD*
- each bidder decides whether (s)he wants to place a higher bid
- when no bidder wants to place a higher bid: the auction ends
- who set the highest bid pays that bid (first-price auction)

Japanese auction

- the prices rises continuosly and bidders drop out until only one bidder remain

English Auction

Open ascending-bid first-price auction

Dutch or Clock Auction

3 volunteers:

- an actioner
- bidder A
- bidder B

Dutch or Clock Auction

Focus on an auctioner that wants to sell a diamond:

- (s)he starts asking if a bidder wants to buy at a high price $1000\,USD$
- If there is no bidder that wants to buy, the auctioner decreases the price repeatedly
- The first bidder that accept to buy wins the auction and pays the bid.

Dutch or Clock Auction

Open descending-bid first-price auction

Dutch or Clock Auction

Examples:

- Fed of NY to sell bonds to primary dealers
- Aalsmeer Flower Acution in the Neatherlands (with a clock that tells the bidders the current price)

Sealed-bid Auction (first-price)

4 volunteers:

- an actioner
- bidder A
- bidder B
- bidder C

Sealed-bid Auction (first-price)

Focus on an auctioner that wants to sell a lighthouse on behalf of the government:

- the minimum bid is set to $20,000 USD$
- each bidder send a bid in a closed envelope to the auctioner
- who set the highest bid pays that bid and wins (first-price auction)

Sealed-bid Auction (second-price)

4 volunteers:

- an actioner
- bidder A
- bidder B
- bidder C

Sealed-bid Auction (second-price)

Focus on an auctioner that wants to sell a bike:

- the minimum bid is set to $50 USD$
- each bidder send a bid in a closed envelope to the auctioner
- who set the highest bid wins and pays the second highest bid (second-price auction)

Sealed-bid Auction (second-price)

example:

- internet-search engines to sell advertising space

Outline

Outline

Definition

- How to behave at each possible situation
- **Weakly dominant**: let you at least as better off than pursuing another strategy for any given choice by the other bidders
- *we consider only auction of a single object with only private value*

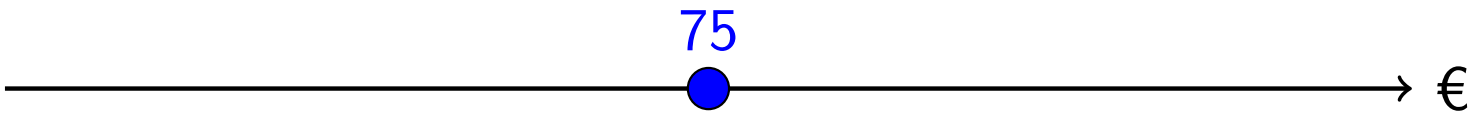
Assumption

- private value is *i.i.d.*
- bidders are risk neutral

Outline

Context

You value the product € 75

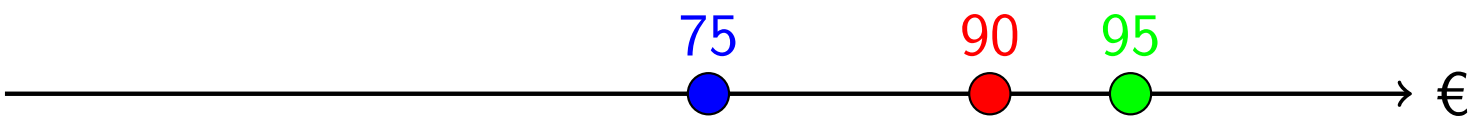


What is your optimal bid?

We have three possibility:

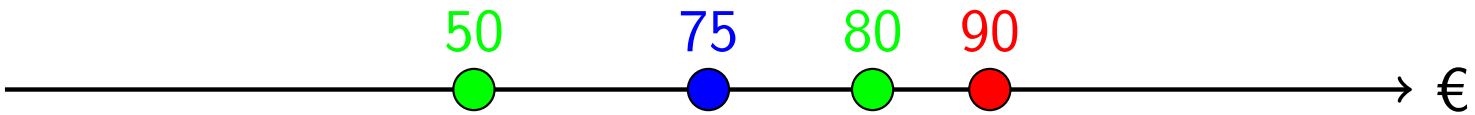
- Make a bid at an amount above € 75 to secure the good (e.g. € 90)
- Make a bid at an amount exactly equal to € 75
- Make a bid at an amount below € 75 to save some money (e.g. € 60)

Make a bid of € 90



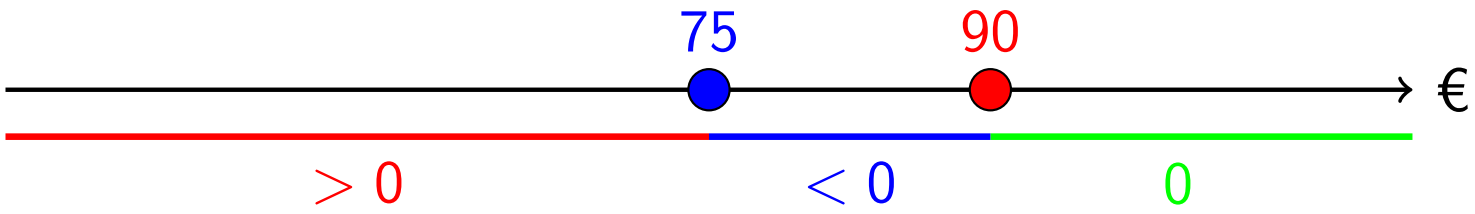
You loose and pay nothing

Make a bid of € 90

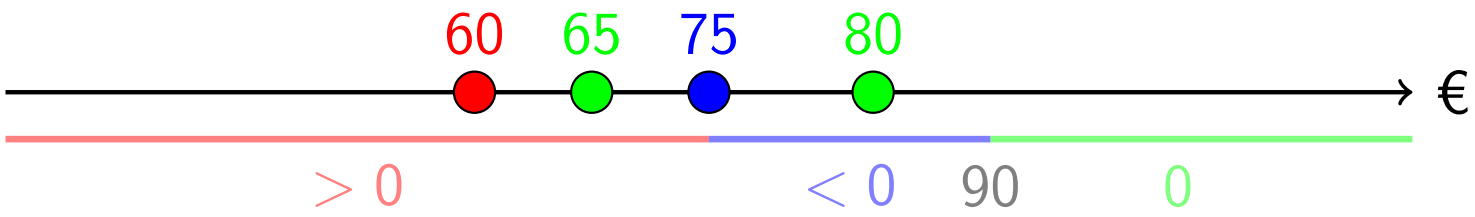


You win and pay € 80 (you loose 75-80=-5) or you pay €50 (you gain 75-50=25)

Make a bid of € 90

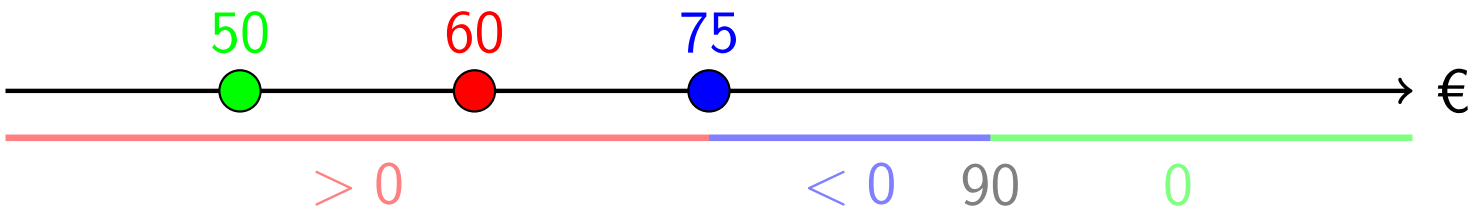


Make a bid of € 60



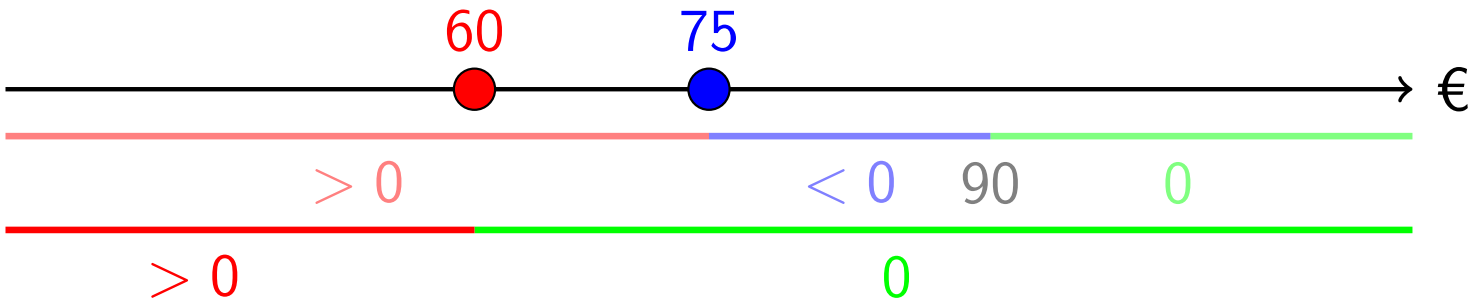
You loose and pay nothing

Make a bid of € 60

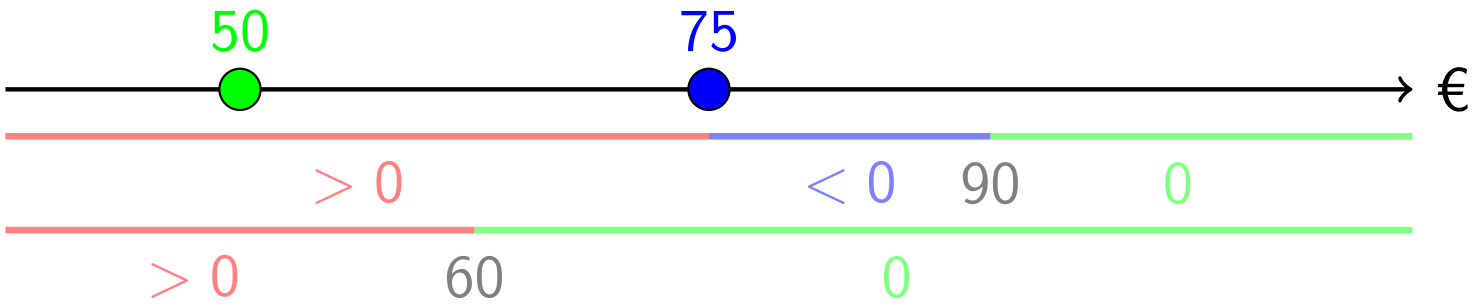


You win and pay €50 (you gain 75-50=25)

Make a bid of € 60

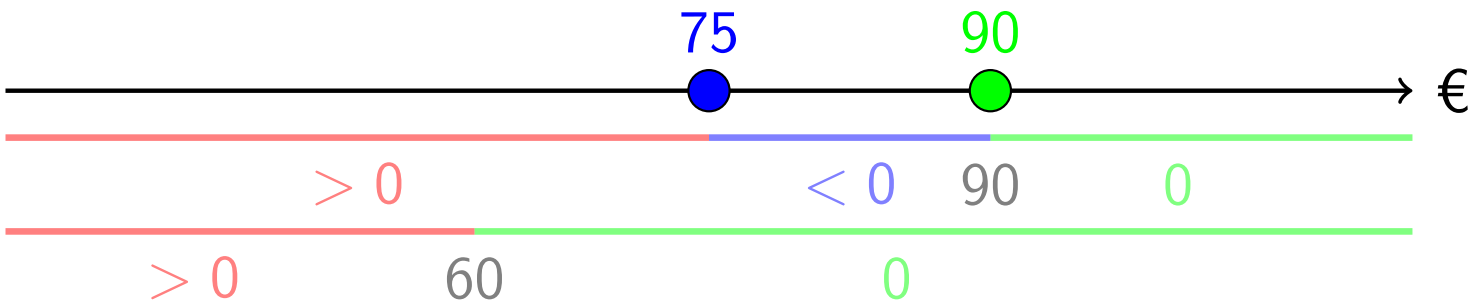


Make a bid of € 75



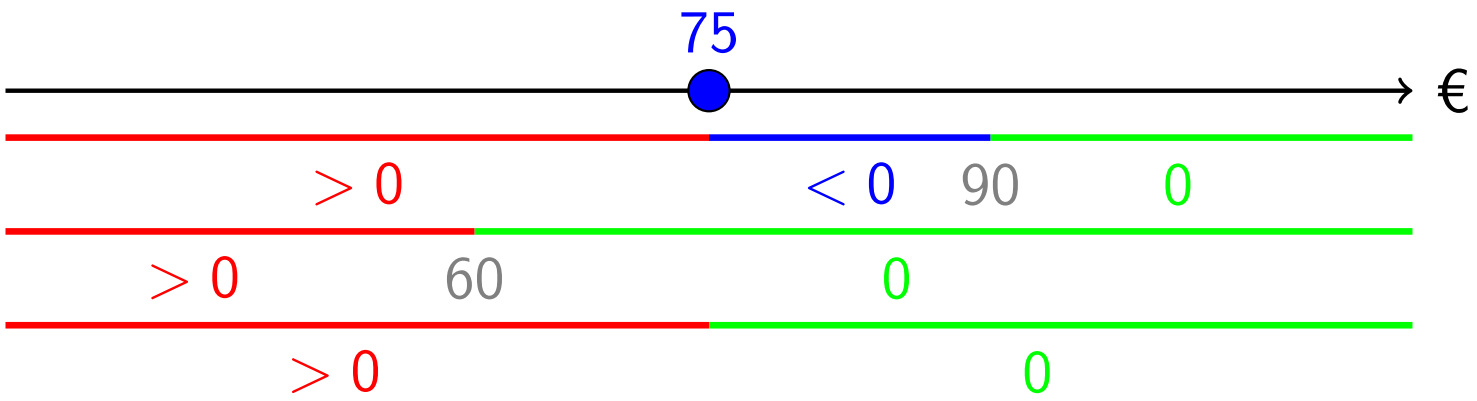
You win and pay €50 (you gain 75-50=25)

Make a bid of € 75



You loose and pay nothing

Make a bid of € 75



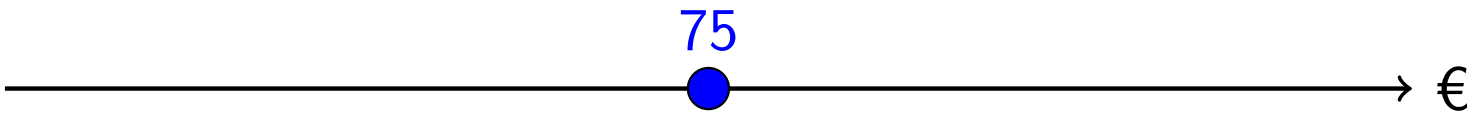
Conclusion

- You will bid exactly your private value!
- The expected revenue is the second highest private value

Outline

Context

You value the product € 75



What is your optimal bid?

We have 2 scenarios:

- the maximum bid (except our one) is above € 75
- the maximum bid (except our one) is below € 75 (e.g. € 60)

the maximum bid (except our one) is below € 75

You have an incentive to increase your bid:

- offer of bidder i: € $\max_{j \neq i} P_j + \varepsilon \approx 60 + \varepsilon$
- positive consumer surplus instead of 0

the maximum bid (except our one) is above € 75

You don't have any incentive to bid anything more than €75 because doing so you will suffer a loss

Conclusion

The outcome is the same as in the sealed bid second price auction

- The expected revenue is the second highest private value (the price at which only one bidder remains in)
- they are strategically equivalent!

Outline

Dutch Auction

The bidder has to decide *a priori* at which price to call out (she will pay that price)

Outline

Sealed bid first price auction

The bidder has to decide *a priori* which price to write in the envelope (she will pay that price)

Conclusion

- Dutch auction and sealed bid first price auction are **strategically equivalent**
- There is no dominant strategy
  - lowering the price:
    - increase the potential gain
    - decrease the probability of winning

Outline

3. Strategy
   - Introduction
   - Sealed bid second price auction
   - English auction
   - Dutch Auction
   - Sealed bid first price auction
   - Revenue Equivalence Theorem

Revenue Equivalence Theorem

It can be proved that the expected revenue for any format of private value auction is the same

Intuition

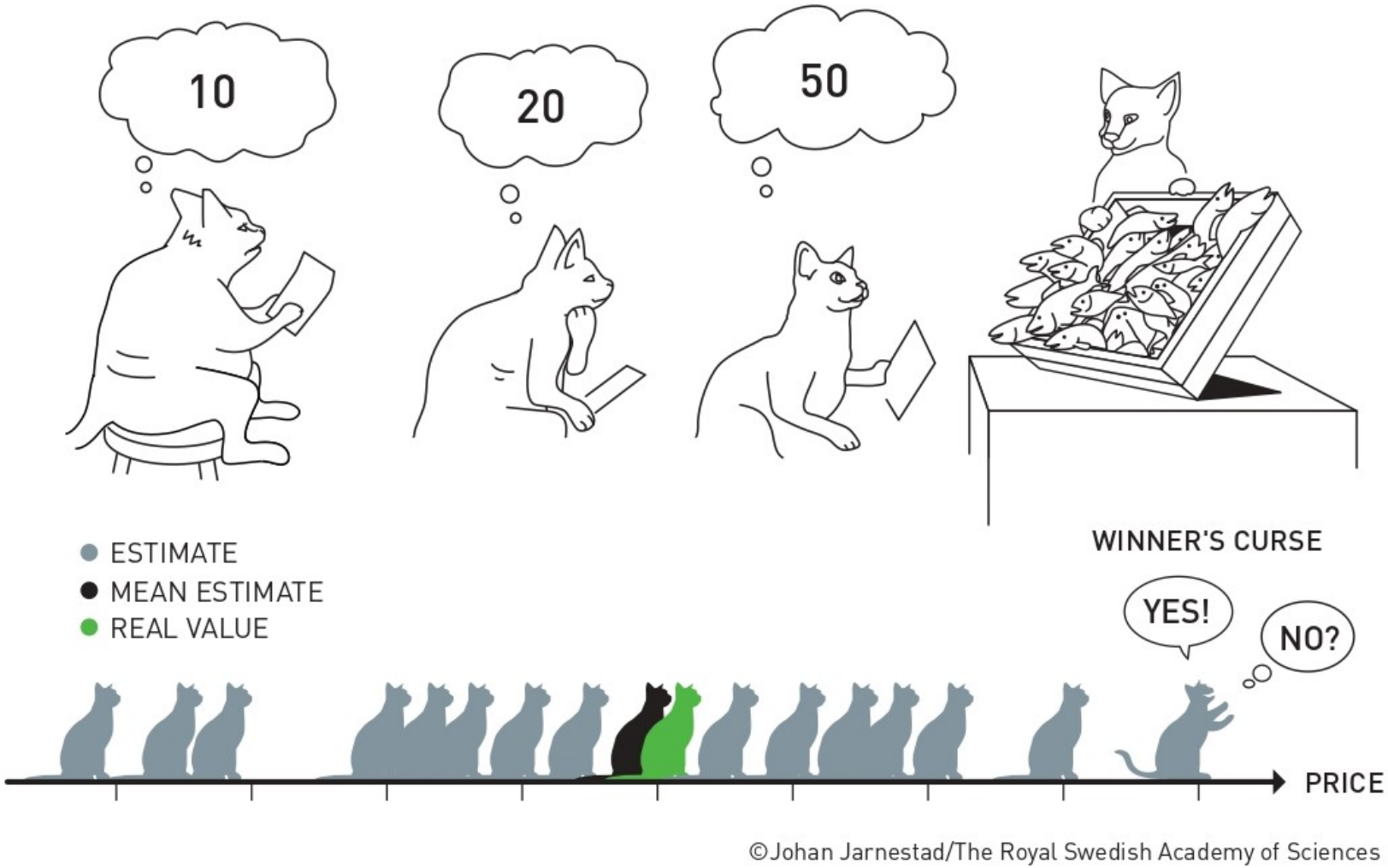
In a first price auction you bid on average the second highest price

Outline

Winner curse

- Focus on auctions with *pure* common value
- Who wins, has evaluated the objects the most
- But the value of the object is unique what differs is private information
- Hence, he has been too optimistic and he has done a bad deal

Winner curse



Formally

- Given the true common value β unique across bidders$i \in I$
- Each bidder observe an idiosyncratic signal that depends on the true value and on a iid noise term:

$$\vartheta_i = \beta + \varepsilon_i$$

$$\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$$

- Hence from the signal extraction problem:

$$E(\beta|\theta_i) = \theta_i = \beta + \varepsilon_i$$

Formally

- We define a bidding function for each bidder $i \in I$:

$$b_i : E(\beta|\theta_i) = \theta_i \mapsto b_i(\theta_i)$$

$$\frac{\partial b_i}{\partial \theta_i} > 0$$

- If i wins:

$$E(\beta|\theta_i, \theta_i > \theta_{-i}) < \theta_i$$

- Hence it is a bad news unless:

$$\varepsilon_{-i} < \varepsilon_i < 0$$

Consequences

- Bid less than under perfect information anticipating that you will pay only in case of victory and so only if:

$$E\{\beta|\theta_i, \theta_i > \theta_{-i}\} < \theta_i$$

- On average $E(\theta_i) = \beta$
- Expected revenue under imperfect information is lower than expected
- Providing information may improve expected revenue:
  - English auction provides information

Bibliography

- Jeffrey M. Perloff (2017). Microeconomics, Theory and Applications with Calculus, Pearson
- Scientifc Background on the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2020
- Paul Klemperer (1999). Auction Theory: A guide to the Literature